Designing police patrol districts on street network

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Abstract

This paper deals with the police districting problem on the street network. Traditionally, design of police patrol sectors is based on grids or census blocks, which may generate districts that are difficult to cover. This problem may be alleviated using a network-based model. This paper formulates a Street Network-based Police Districting Problem (SNPDP) model, which simultaneously considers the connectivity, workload efficiency and balance of districts. An algorithm framework is also proposed to solve this problem. This provides a conceptual based toward finding suitable districting plans for patrol sectors.

Keywords: police districting problem, street network, connectivity, efficiency, balance, local search.

1 Introduction

The Police Districting Problem concerns the efficient design of the patrol sectors with respect to performance attributes such as workload, response time, topology, etc. Traditionally, the police districts were manually drawn by police officers on a road map by following the main streets in the area without accomplishing workload balance or geographic compactness (Bruce 2009). In recent decades, with the advance in Geographic Information System and computer technology, automatic methods for defining police districts have gained increasing attention among police departments (Wang 2012). Since the first paper on PDP by Mitchell (Mitchell 1972), various mathematical optimisation models for PDP have been proposed, and these models have considerably improved on hand-made district designs in terms of balanced workload distribution among districts.

So far, most models for PDP use as basic units the grids or the census blocks in the area. For convenience, they are called block-based PDP in this paper. Using a grid structure in districting problems simplifies the problem definition and computation. On the other hands, using census blocks as the atomic units of police districting is desirable, as it allows easy access to demographic data and it is readily suitable for use by other agencies (Sarac et al. 1999). As an example, a multi-criteria PDP model has been developed on the basis of grids and census blocks respectively (Camacho-Collados et al. 2015; Liberatore & Camacho-Collados 2016).

While the block-based PDP have shown promising results, using the street segments as the basic units may be more meaningful for the design of patrol sectors. There are a number of reasons why street network-based models are appropriate for the PDP. The first is simply that the features of street network influence both the long-term crime risk and the short-term dynamics of crime behaviours, so that streets represent a meaningful units for crime prevention than grids or census blocks. In addition, it has seen a trend of studying and predicting crime at the street level. The network-based PDP is compatible with this line of research and hence can readily make use of the results of network-based crime prediction. Moreover, as street network fundamentally influences the movement of police officers, network-based models would produce districting plans of better usability than block-based alternatives. Generally, since grid squares or census blocks could intersect physical barriers and contain unconnected street segments, they are less suitable for operational deployment than street segments. Furthermore, using street segments as units can prevent area unit problems (e.g., grid size) (Camacho-Collados et al. 2015), which is difficult to solve.

The contribution of this research are as follows. First, we introduce a model for SNPDP that use the streets as basic units. To our knowledge, this is the first street network-based PDP model in literature. We redefine the concepts and constraints from block-based PDP in the new context. Second, we propose a local search-based algorithm framework to solve the SNPDP.

The rest of the paper is organised as follow. In section 3, we present the street network-based PDP model. In section 4, we propose an algorithm framework to solve the SNPDP. Finally, we conclude with some remarks and directions for future research.

2 A Street Network-based Police Districting Problem

We define the SNPDP on the street network. Our focus is on the foot patrol where the one-way streets are not a restriction, and hence the underlying street network is modelled by an undirected graph $G=(V,E)$, V corresponding to the street intersections or dead ends and E the set of streets. We also assume that G is connected. If G is not connected, each connected components of G has to be considered separately. Here we provide an overview of the components in the SNPDP model. The terminology in the SNPDP is illustrated in Table 1.

Table 1. Terminology in SNPDP

Term	Meaning	Attributes
Area	the street network	
Basic units	Street segments	Length; Crime risk
District	a connected part of the area	Area; Crime Risk; Diameter
Districting plan	the partition of the area into districts	
Districting problem	The problem of finding the optimal districting plan	Constraints; Objective function

2.1 Basic units

The basic units correspond to streets in the network. Each street e has two attributes, namely the length $a(e)$ and the crime risk $r(e)$. The crime risk on streets can be predicted by predictive crime mapping (Rosser et al. 2016). Each basic unit is specified by the geographic coordinates of the two end points. Here we need to define the distance between two nodes and two edges of G. The shortest distance between nodes $v_i, v_j \in V$ is denoted by $d(v_i,v_j)$. Given two edges $e_1=(v_1, v_2)$ and $e_2=(v_3, v_4)$, the shortest distance $d(e_1, e_2)$ is defined as $\min\{d(v_1, v_3), d(v_1, v_4), d(v_2, v_3), d(v_2, v_4)\}$.

2.2 Districts

A district D_i is a set of basic units, and is a connected sub-graph of E. Each district has three essential attributes, which are adapted from the multi-criteria PDP (Camacho-Collados et al. 2015) and are converted into dimensionless ratios in order to be comparable.

- Area, A

This attribute measures the size of the area that an agent should patrol. The area measure is the ratio of the total length of streets in the district to the whole area

$$A(D_i) = \frac{\sum_{e \in D_i} a(e)}{\sum_{e \in E} a(e)} \tag{1}$$

- Risk, R

This attribute measures the total risk of the district, i.e. the sum of the risks associated with the basic units in the district. The risk measure is the ratio of the total risk of the district to the whole area:

$$R(D_i) = \frac{\sum_{e \in D_i} r(e)}{\sum_{e \in E} r(e)} \tag{2}$$

- Diameter, Dt

The diameter of a district is defined as the maximum distance between two basic units in this district. Intuitively, a compact district has small diameter. Moreover, the diameter approximates the maximum distance a patroller has to travel in case of a call for service. Therefore, small diameters implicate low response time. The diameter measure is the ratio of the district diameter to the maximum distance between two edges in the whole area:

$$Dt(D_i) = \frac{\max\limits_{a,b \in D_i} \{d(a,a)\}}{\max\limits_{a,b \in E} \{d(a,b)\}} \tag{3}$$

By combining the dimensionless measures we may define the workload of a district. The relative importance of the attributes can be expressed by the weights: w_A, w_R, and w_{Dt}. A larger weight corresponds to higher importance of an attribute. Hence, we define the workload as the weighted sum of the three attributes:

$$W(D_i) = w_A \cdot A(D_i) + w_R \cdot R(D_i) + w_{Dt} \cdot Dt(D_i) \tag{4}$$

2.3 Constraints

A districting plan is a set of p districts $P = \{D_1, D_2, \ldots, D_p\}$, where p is the given number of districts. The aim of the problem is to partition all basic units into p district that are connected and balanced. Based on previous research (Camacho-Collados et al. 2015), we identify the constraints of the street-network PDP in the following.

- Each basic unit must belong to exactly one district.
- The subgraph of G induced by each district must be connected. In other words, any two nodes belonging to a district must be reachable within the district.
- The districting plan should be as efficient as possible, meaning that the sum of workloads of all districts should be small.
- The districts should be homogeneous and balanced in terms of workload, meaning that the maximum or worst workload of districts should be small.

2.4 Objective function

The constraints discussed may be contradictory and thus needs a trade-off. For example, an increase in the balance of the workloads may increase the total workloads and reduce the efficiency. Therefore, an objective function is defined that considers both the efficiency and homogeneity:

$$obj(P) = \lambda \cdot \max_{D \epsilon P}\{W(D)\} + (1 - \lambda) \cdot \frac{\sum_{D \epsilon P} W(D)}{p} \tag{5}$$

Where \lambda $\in$[0,1] and represents the preference of the decision maker on the two factors. The term $\max_{D \epsilon P}\{W(D)\}$ expresses the maximum or worst workload and the balance of workloads, while the term $\frac{\sum_{D \epsilon P} W(D)}{p}$ is the average workloads.

2.5 Problem formulation

Therefore, the street network-based PDP is formulated as

$$\min obj(P) \tag{6}$$

$$s.t. \bigcup_{D \epsilon P} D = E \tag{7}$$

$$A \cap B = \emptyset, \forall A, B \in P, A \neq B \tag{8}$$

$$|P| = p \tag{9}$$

$$Connect(D) = 1, \forall D \in P \tag{10}$$

The model minimises the objective function (6). Constraints (7) and (8) represent that the districts cover the edge set and are pairwise disjoint. Constraint (9) enforces the number of districts to be exactly p, which is given by the decision maker. Constraint (10) requires that each district should be connected.

3 An algorithm framework for the SNPDP

Here we propose an algorithm framework to approximately solve the SNPDP, and the flowchart of the algorithm is illustrated in Figure 1. It follows a multi-start procedure (Martí 2003). Multi-start is a diversification method to better explore different parts of the solution space. It enforces the search to start multiple times from different points. In each iteration, an initial solution is generated, by using a variant of a random greedy algorithm (Liberatore & Camacho-Collados 2016) or the Karlsruhe High Quality Partitioning method (Christian 2013). The latter may be more efficient for problems with hundreds of basic units. Then, the solution is improved using a local search method, such as Simple Hill Climbing or tabu search (Glover 1989). The steps are repeated until the termination criteria is met.

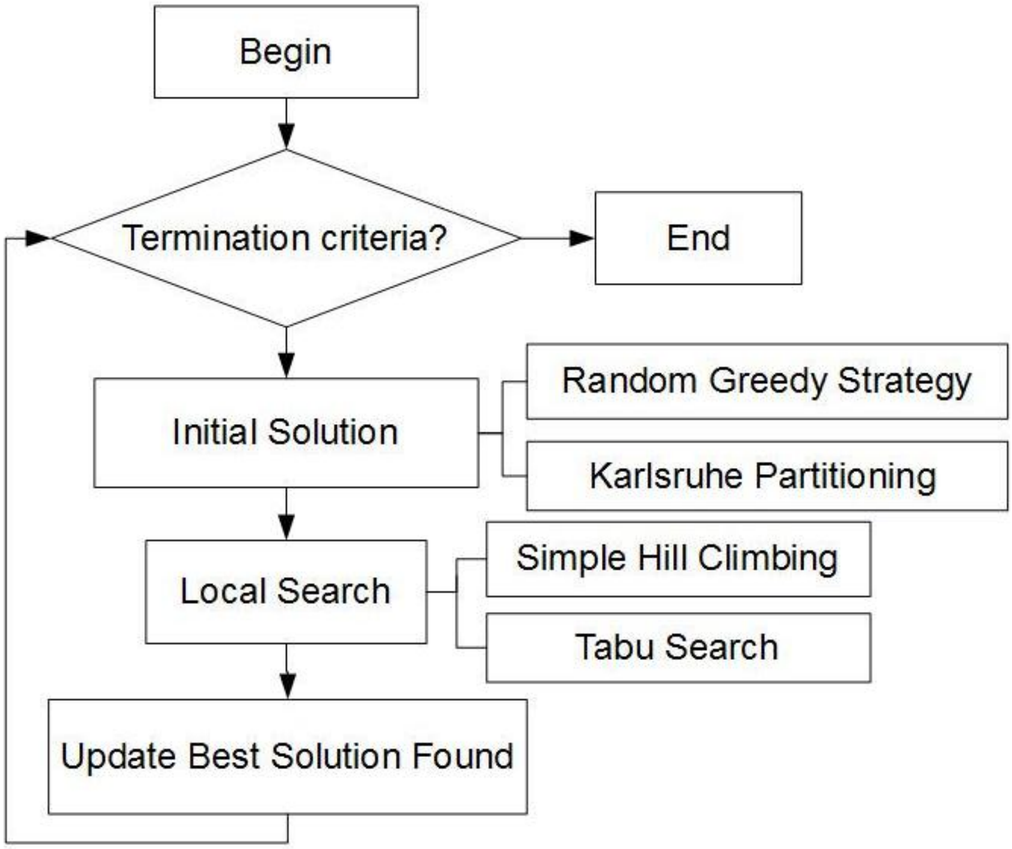

Figure 1. Flowchart of the algorithm for SNPDP

4 Conclusions

In this paper, we present an innovative Police Districting Problem based on street network (SNPDP), which has several advantages over block-based PDP. We define the various constraints and a multi-criteria objective function of SNPDP. An algorithm is proposed to solve SNPDP, which uses multi-start and local search methods to iteratively identify better solutions. Further research will focus on implementing, testing and validating the algorithm.

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