Determination of Spatially Distributed Velocity for Flow Routing

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Abstract

The physically-based distributed hydrological models play an important role in watershed hydrology. Physical simulation of flow routing depends largely on the field distribution of flow velocity.Terrain analysis studies have previously focused more on the delineation of geomorphologic structures rather than the description of hydraulic factors. As affected by multiple uncertain factors including topography, soil properties and water depth in channel, the determination of overland flow velocity is limited.This study proposed a statistical method to generate a field map of spatially distributed flow velocity from a grid structure. The upstream drainage area for each grid cell was involved to represent the hydraulic radius in Manning's equation. The weight of hydraulic factor was calibrated by adjusting the impact on flow velocity. Case study of flow routing was undertaken to validate the field distribution of flow velocity. The results indicate that the proposed method can reasonably estimate flow velocity field distribution,and the calibrated value of weighting coefficient of hydraulic factor can produce acceptable unit hydrograph for rainfall-runoff processes.

Keywords:flow routing, terrain analysis, digital elevation model, flow velocity

1. Introduction

Watershed hydrology is the study of the water cycle from point view of drainage basin. As one of the most important hydrologicalcomponents, the process of rainfall-runoff is largely governed by spatially distributed factors such as soil type, vegetation coverage and topographic relief (Beven 2012). Numerous watershed models have been developed to describe the process details, mainly including where to go when it rains (hydrologic factors), what path to take when it flows (geomorphologic factors), and how long to remain in the basin (hydraulic factors) (Hewlett and Hibbert, 1967; McDonnell, 2003).Traditional lumped models rely on historical observations and lack explanatory power(Devia et al., 2015).With the increasing application of remotely sensed (RS) data and geographical information system (GIS) techniques, terrain analysis method shave contributed to the growing physically-based spatially distributed hydrological models. Many studies have focused on deriving geomorphologic features from gridded digital elevation model (DEM) (O'Callaghan and Mark, 1984; Tarboton, 1991).However, there are few studies on describing hydraulic factors such as flow velocity. That is mainly because the flow velocity is affected by multiple factorsincluding topography, surface roughness and water depth(Maidment et al., 1996; Rui et al., 2008).This study, therefore, proposed an empirical method to determine the velocity field of surface flow based on a terrain model. The D8 algorithm was used to calculate upstream accumulation area and slope for each cell in a gridded DEM(O'Callaghan, 1984; Fairfield, 1991).The Manning's equation was further modified to derive flow velocity from the upstream area and slope (Maidment et al., 1996).Experiments of rainfall-runoff modellingwere undertaken to validate the velocity field of surface flow. The results indicate that the proposed method of generating field map for flow velocity can produce more acceptable outcomes.

2. Method

The simulation of flow routing relies on the spatial distributions of rainfall, flow path and velocity.Theunit hydrograph approach proposed by Sherman in 1932useaconstant flow velocity to a basin, where the runoff responseis linear to the rainfall intensity. Rodriguez-Iturbe and Valdés (1979)investigated the influences of geomorphologicfactors on unit hydrographbycontrollingthe Horton's parameters and the constant flow velocity of the basin. The experimental results indicated that the hydrologic response is intimately related to geomorphologic parameters and flow velocity. Maidment et al. (1996) improved the unit hydrograph approach by modifying the Manning's equation. The Manning's hydraulic radius was replacedby upstream area on each grid cell. It turned out that the outcomes had been improved by the field distribution of flow velocity. However, there is reason to believe that the weighting coefficient of upstream area on flow velocity could vary from watershed to watershed. This study, therefore,introduces an empirical method to determine the weighting coefficient by adjusting the influences of Manning's factors on flow velocity.

As demonstrated in Figure 1, a grid DEM(a) is used to derive cell flow direction (b) and flow path (c) (O'Callaghan, 1984; Fairfield, 1991), then a three-step framework is established to (d)determine field distributionof flow velocity, (e) calculate time duration of flow accumulation, and (f) simulate the process of flow routing.

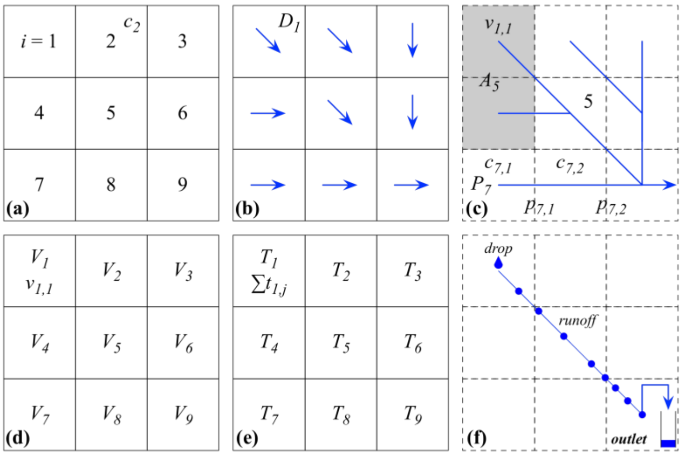

Figure 1: The generation of flow paths and simulation of flow routing based on DEM.

The process of flow routingdepends on both geomorphic and hydraulic factors. In this study, as listed in Table 1, these factors are distinguished by cell-based and path-based parameters. First, hydrologic factors such asflow direction D_i and accumulation V_i are derived for grid cells by the D8 algorithm(O'Callaghan, 1984). Second, Hydraulic factors such as flow velocity $v_{i,j}$ and time $t_{i,j}$ are calculated for flow path segments by the modified Manning's equation. Third, the total flow time T_i from cell c_i to the basin outlet is assigned back to each grid cell by summing up the segment flow time $t_{i,j}$ along the flow path. Fourth,hourly rainfall intensities $h_i(t)$ are allocated into each grid cell c_i, and the outlet discharges $Q(t)$ can then be calculated over time.

Table 1: The determination ofhydrologic and hydraulic factors for flow routing.

Object	Factor	Symbol	Calculation
Cell	grid cell	c_i	$i = 1, 2, \cdots, N_c$
	catchment area	A	$A = N_c \cdot A_c$
	cell steepest slope	S_i	$S_i = \Delta h_i / l_i$
	cell flow direction	D_i	$D_i = direction(c_i)$
	cell upstream area	A_i	$A_i = A_c \cdot \sum_{j=1}^{N_c} c_{j \to x \to i}$
	cell flow velocity	V_i	$V_i = \alpha {A_i}^{\beta} S_i^{0.5}$
	cell flow time	T_i	$T_i = \sum_{j=1}^{N_i} t_{i,j}$
	runoff intensity	$h_i(t)$	*interpolation method*
	outletdischarge	$Q_i(t)$	$Q(t) = \sum_i^{N_c} \int_0^t A_i \cdot h_i(t)\, dt$
Path	path to outlet	P_i	$P_i = \{p_{i,j}\}(j = 1, 2, \cdots, N_i)$
	path length	L_i	$L_i = \sum_{j=1}^{N_i} l_{i,j}$
	path segment	$p_{i,j}$	$p_{i,j} = \{c_{i,j}, c_{i,j+1}\}$
	segment length	$l_{i,j}$	$l_{i,j} = distance(c_{i,j}, c_{i,j+1})$
	segment velocity	$v_{i,j}$	$v_{i,j} = V_{i,j}$
	segment time	$t_{i,j}$	$t_{i,j} = l_{i,j} / v_{i,j}$

2.1 Determination of flow velocity

The flow velocity is believed to relate to the bed topography, soil properties and water depth. The Manning's equation has been widely used to calculate the flow velocity in channels:

$$V = {}^{1}/_{n} \cdot R^{\frac{2}{3}} \cdot S^{\frac{1}{2}} \qquad \text{Equation 1}$$

where V is the estimation of flow velocity, n is the surface roughness coefficient representing the influence of soil prosperities on flow velocity, R is the hydraulic radius representing the influence of hydraulic factor on flow velocity, and S is the water surface slope representing the influence of geomorphic factor on flow velocity. Because the hydraulic radius R is related to the upstream drainage area A, Equation 1 can be written as:

$$V = \alpha \cdot A^{\beta} \cdot S^{0.5} \qquad \text{Equation 2}$$

where α is a constant coefficient for standardization because of the uncertainty in dimension, and β is an undetermined weighting coefficient representing the influence of upstream area on flow velocity. Maidment et al. (1996) set the value of β to 0.5, and expressed the factor α as:

$$\alpha = V_m / [A^{0.5} \cdot S^{0.5}]_m \qquad \text{Equation 3}$$

where V_m is the mean flow velocity in the basin, while $[A^{0.5} \cdot S^{0.5}]_{\mathrm{m}}$ is the mean value of the calculation of $A^{0.5} \cdot S^{0.5}$. A constant value of 0.5 was set to the weighting coefficient of upstream area. However, there is reason to believe that the weighting coefficient β should be adjustable. This study introduces an adjusting function to determine the value of β:

$$\beta = \{\beta |\ cor(V, A)/cor(V, S) = 1\} \qquad \text{Equation 4}$$

where $cor\ (X, Y)$ means the correlation coefficient between variable X and variable Y. The weighting coefficient β affects the influence of the hydraulic factor on the flow velocity. The higher the value of β, the greater impact on flow velocity will be. The function of Equation 4 is to ensure the hydraulic factor A and the topographic factor S have the same impact on flow velocity.

2.2 Accumulation of flow time

Based on the field distribution of flow velocity, the mean velocity $v_{i,j}$ of each segment $p_{i,j}$ on path P_i is approximated by the flow velocity V_i on the starting cell c_i. The flow time T_i from each grid cell c_i to its outlet can then be accumulated by equation 5:

$$T_i = \sum_{j=1}^{N_i} l_{i,j} / v_{i,j} \qquad \text{Equation 5}$$

where $i = 1, 2, \ldots, N_c$ means the index of grid cells in basin, $j = 1, 2, \ldots, N_i$ represents the index of grid cells on the path P_i starting from cell c_i, and $l_{i,j}$ means the length of the j-th segment on path P_i.

2.3 Simulation of flow routing

Based on the field distribution of flow time T_i and runoff intensity $h_i(t)$, the discharge $Q(t)$ at the cross section of basin outlet can be estimated by Equation 6:

$$Q(t) = \sum_{i}^{N_c} A(c_i) \cdot h_i(t - T_i) \qquad \text{Equation 6}$$

where $A(c_i)$ means the surface area of the grid cell c_i, T_i means the total flow time from this grid cell to the basin outlet, and $h_i(t - T_i)$ means the runoff intensity over grid cell c_i at time $t - T_i$.

The production of surface runoff is related to the rainfall intensity, land use type and soil properties. This paper considers only the process of flow routing, then the process of runoff generation is not discussed. The runoff generation is estimated from the rainfall intensities by the Soil and Water Assessment Tool (SWAT) model (Arnold et al., 1998). According to Equation 6, surface runoff generated on the grid cell c_i at time $t - T_i$ will reach the basin outlet after the duration of T_i. As a result, the discharge at the cross section of basin outlet at time t are accumulated.

3. Case Study

A case study has been conducted based on the 30-meters Advanced Space borne Thermal Emission and Reflection Radiometer (ASTER) DEM data over a small catchment, as shown in Figure 2 (a), the study area consists of 5322 grid cells, covering an area of about 5 square kilometres. Figure 2 (b) and (c) show the spatial distributions of upstream drainage area and slope of grid cell. The hourly Global Satellite Mapping of Precipitation (GSMaP) data in July 2016 over the study area were acquired for the simulation of flow routing.

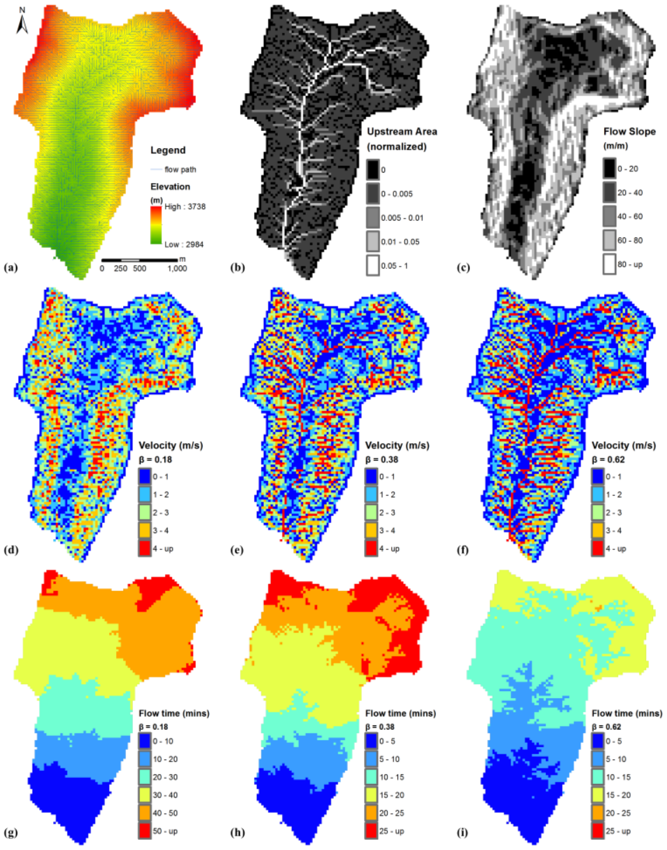

Figure 2: The spatially distributed flow velocity and time for the grid cells.

To determine the appropriate value for the weighting coefficient β, as shown in Figure 3 (a), the correlation coefficients $r$between the flow velocity and the two factors are illustrated by adjusting the value of $\beta$from 0 to 1. As the weighting coefficient β increases, the impact of upstream area becomes greater, and the influence of slope on flow velocity decreases. When the value of β is between 0.18 and 0.62, the two factors have a positive correlation, and the two factors have the same impact when β is set to a value of 0.38. In this study, as shown in Figure 1 (d), (e) and (f), the value of 0.18, 0.38 and 0.62 is set to β to calculate the flow velocity respectively. Figure 1 (g), (h) and (i) show the corresponding field map off low time. Figure 3 (b) and (c) demonstrate the probability densities of the three groups of flow velocity and time on grid cell.

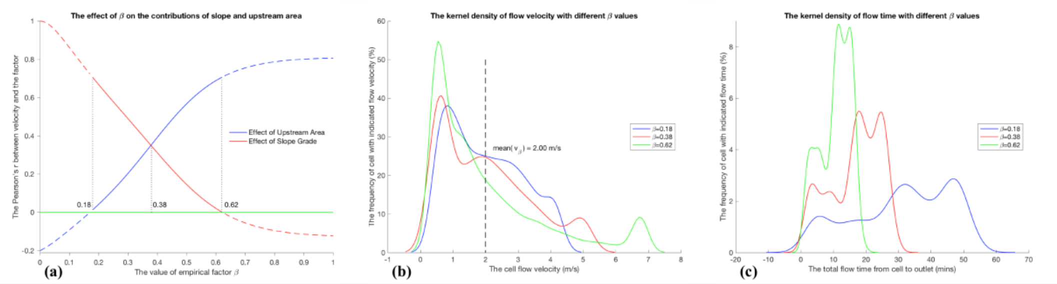

Figure 3: The determination of weighting coefficient for flow velocity estimation.

In this study, the mean flow velocity $V_m$is fixed to 2 m/s, and the weighting coefficient β is set to 0.18, 0.38 and 0.62. According to the field distributions of flow velocity and total flow time in Figure 2 and 3, the greater the value of β, the faster flow velocity and the shorter flow time will be. For example, if the value of β is set to 0.18, the maximum flow velocity on grid cell is about 5 m/s and the longest flow time is about 65 minutes.If the value is set to 0.62, the values are respectively7.5m/s and 22 minutes.

Flow routing program has been developed andapplied to predict discharges at the basin outletaccording to Equation 6. The simulated basin discharges are compared to the outcomes of SWAT model (Arnold et al., 1998).Figure 4 shows the unit hydrographs of the three groups of flow time distribution. The time delay in peak discharge is related to probability distribution of flow time. Simulation results confirm that the longer the total flow time, the longer delay is.Besides, the Nash and Sutcliffe efficiency (NSE) is used as an indicator of the goodness of fit between the simulated discharges and the SWAT outputs.Comparisons results indicate that the proposed method of determining field distribution of flow velocity can achieve relatively better outcomes.

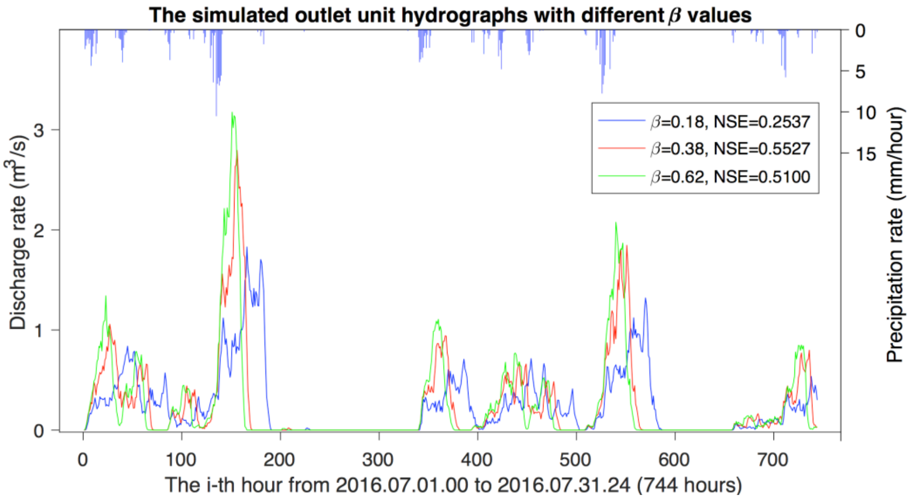

Figure 4: The simulated discharge dynamics at the basin outlet under different velocity distribution.

4. Conclusion

This study proposed an empirical approach for the determination of field distribution offlow velocity. The commonly used Manning's equation is further modified for easier use by replacing the hydraulic radius with upstream area. The modified approach has two undetermined parameters, the mean flow velocity in basin and an adjustable weighting coefficient. The mean flow velocity may be determined by field measurement, and the weighting coefficient is estimated by adjusting influences of the hydraulic factor (upstream area) and topographic factor (slope) on flow velocity.

Experiments of flow routing were done based on spatially distributed rainfall inputs and field distribution of flow velocity. The experimental results show that the proposed approach can achieve better simulation outcomes for the process of flow routing. Further studies are required to validate the proposed approach with field measurements over a range of catchments. Besides, this study only considers the process of flow routing,hence underlying conditions such as plant cover, soil type and land use are not discussed. Further attention should be paid to enhance the proposed approach.

5. Acknowledgments

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